QF 5206 Topics on Quantitative Finance
Group Project report

Replication: Bayesian Adaptive Trading with a Daily Cycle

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1. Introduction

In this report, we replicate the paper" Bayesian Adaptive Trading with a Daily Cycle" (Robert Almgren and Julian Lorenz) from the Journal of Trading, 2006 and apply the model introduced in the paper for our project. In the article, it showed a model for price dynamic and optimal trading that explicitly includes the daily trading cycle and the trader's attempt to learn the targets of other market participants. This is in contrast to most current models of optimal trading strategies that consider time as an undifferentiated continuum, and that consider other traders as a collection of random noise sources.

This paper has several assumptions and features. First of all, the authors consider that informed participants will use all available information to compete with each other. Secondly, they use a popular execution algorithms that adapt to changes in price of the asset being traded, either by accelerating execution when the price moves in the trader's favor, or conversely.

According to the paper, there is an underlying drift factor caused by the net positions being executed by other institutional investors. This factor is approximately constant throughout the day because other traders execute across the entire day. Thus price increases in the early part of the day suggest that this factor is positive, which suggests that prices will continue to increase throughout the day.

The daily cycle is an essential feature of this model. Many institutional participants make investment decisions overnight and implement them through the following trading day. Within each day, the morning is different from the afternoon because an intelligent trader will spend the early trading hours collecting information to trade in the rest of the day.

Moreover, there is a very important feature of constraints on trade direction, the trader must never sell as part of a buy program even if this would yield lower expected costs (or give an expected profit) because of anticipated negative drift in the price.

In the rest of this report, we first introduce the price model which is talked in the paper and discuss the trading and price impact. Section 3 describes the optimal trading strategies under some assumptions for the model. Section 4 then reports the simulation results and provides some analysis. At last, we will highlight some difficulties of doing empirical data study and some remarks for our report.

2. Price Model Including Bayesian Update

In this part, we will introduce the Bayesian price model in Almgren and Lorenz (2006).

Trading in a single asset whose price is S(t), obeying an arithmetic random walk

$$S(t) = S_0 + \alpha t + \sigma B(t) \quad \text{for} \quad t \geq 0 \tag{1}$$

where $B(t)$ is standard Brownian motion, σ is an absolute volatility and α a drift.

The interpretation according to the paper is that volatility comes from the activity of the "uninformed" traders, whose advantage behavior can be predicted reasonably well. Mathematically, we assume that the value of σ is known precisely. (For a Brownian process, σ can be estimated arbitrarily precisely from an arbitrarily short observation of the process). The drift is as coming from the activity of other institutional traders, who have made trade decisions before the market opens and who expect to execute these trades throughout the day. If these decisions are in the aggregate weighted to buys, then this well cause positive price pressure and an upwards drift; the converse is true for overall net selling. Moreover, the drift α is assumed as constant throughout the day, but its value cannot be known. Therefore, at the beginning of the day, we have a prior belief

$$\alpha \sim N(\bar{\alpha}, v^2)$$

which will be updated using price observations during the day. There are thus two sources of randomness in the problem: the continuous Brownian motion representing the uninformed traders and the single drift coefficient representing the constant trading of the large traders.

2.1. Bayesian Inference

Intuitively, as the trader observes prices from the beginning on the day onwards, he or she starts to get a feeling for the day's overall flow.

Mathematically, at time t we know the stock price trajectory $S(\tau)$ for $0 \le \tau \le t$. In fact, all of our information about the drift comes from the final value $S(t)$. Conditional on the value of α, the distribution of $S(t)$ is

$$S(t) - S_0 \sim N(\alpha t, \sigma^2 t),$$

and after some calculation we find the unconditional distribution.

$$S(t) - S_0 \sim N(\bar{\alpha} t, (\sigma^2 + v^2 t)t)$$

then use the Bayes' rule

$$\Pr(\alpha \mid S(t)) = \frac{\Pr(S(t) \mid \alpha) \cdot \Pr(\alpha)}{\Pr(S(t))}$$

to obtain the posterior conditional distribution

$$\alpha \sim N\left(\frac{\bar{\alpha}\sigma^2 + v^2(S(t) - S_0)}{\sigma^2 + v^2 t}, \frac{\sigma^2}{\sigma^2 + v^2 t} v^2\right). \qquad (2)$$

This represents out best estimate of the true drift α, as well as out uncertainty in this estimate, based on the combination of our prior belief with price information observed to time t.

2.2. Trading and Price Impact

The trader has an order of X shares, which begins at time $t=0$ and must be completed by time $t = T < \infty$. For concreteness, we shall suppose $X > 0$ and interpret this as a buy order. A trading trajectory is a function $X(t)$ with $X(0) = X$ and $X(T) = 0$, representing the number of shares remaining to buy at time t. The corresponding trading rate is $v(t) = -dx/dt$. We shall require that $v(t) \geq 0$ for all t, so that the program never sells as part of a buy order. Together with the endpoint constraints, this requires $0 \leq x(t) \leq X$, but it may also be binding in the interior of this region.

According to Almgren and Lorenz (2006), we use a linear market impact function for simplicity. Thus, the actual execution price is

$$S(t) = S(t) + \eta v(t),\ \eta > 0,$$

where η is the coefficient of temporary market impact.

The implementation shortfall C is the total cost of executing the buy program relative to the initial value,

$$C = \int_0^T S(t)v(t)\,\mathrm{d}t - XS_0$$
$$= \sigma\int_0^T x(t)dB(t) + \eta\int_0^T v(t)^2 dt + \alpha\int_0^T x(t)dt \qquad (3)$$

Here α is the true drift that determines the cost whether or not known its value. C is a random variable, both because the price $S(t)$ is random, and because the trading strategy $v(t)$ may be adaptive to S.

3. Optimal Trading Strategies

3.1. Trading Goal

In this part, we consider the question of what trading strategies are optimal, given the above model for price evolution and market impact on the basis of Almgren and Lorenz (2006). In the model, risk-averse trading strategies can behave strangely in time even in the classic framework, depending on the precise formulation of the mean-variance tradeoff. In this case, the problem is complicated by the need to account for the variance in the estimation of α. The authors have obtained partial solutions for the risk-averse problem but their complexity obscures the underlying structure.

Therefore, we have to focus on the drift, which is the most important new aspect of this problem, and we neglect risk aversion; we seek to minimize only the expectation of trading cost. That means

we assume that the pressure to complete the trade quickly comes primarily from a desire to capture the price motion expressed by the drift α, and it is this effect that must be balanced against the desire to reduce market impact costs by trading slowly.

In order to support this description, we shall generally assume that the original buy decision was made because the trader's belief $\bar{\alpha}$ is positive ($\bar{\alpha} > 0$). We then expect $\alpha > 0$ in (3), and the term $\int \alpha x(t) dt$ is a positive cost. The true value of α may have negative value, or intermediate price movements may cause us to form a negative estimate. Because our point of view is that of a broker/dealer executing an agency trade, we shall always require that the trade be completed by $t = T$, unless the instructions are modified.

For any deterministic trajectory $x(t)$ specified at time $t = 0$, C is a Gaussian variable. Conditional on the true value of α, it has expected value

$$E(C) = \eta \int_0^T v(t)^2 dt + \alpha \int_0^T x(t) dt \quad \text{conditional on } \alpha \tag{4}$$

From (2), we know our best estimate for the value of α at time t is

$$\alpha_*(t, S) = \frac{\bar{\alpha}\sigma^2 + \nu^2 (S - S_0)}{\sigma^2 + \nu^2 t} \tag{5}$$

where $S = S(t)$. Because the expectation (4) is linear in α, we may simply plug the expected value α_* to see that, conditional on the

information available at time t, the expected cost of remaining program is

$$E(t,S,x(t),\{x(\tau)\}) = \eta\int_t^T v(\tau)^2 d\tau + \alpha\int_t^T x(\tau)d\tau \qquad (6)$$

On the left, t is current time, $x(t)$ is the number of shares currently remaining to buy, S is the current stock price, and $\{x(\tau)\}$ denotes the liquidation strategy that will be used on the remaining time $t \le \tau \le T$.

Our trading goal is to choose the remaining strategy to minimize this expected cost: determine $x(\tau)$ for $t \le \tau \le T$ such that

$$\min_{\{x(\tau)\}} E(t,S,x(t),\{x(\tau)\}) \qquad (7)$$

To solve this problem, we shall assume that the drift estimate $\alpha_*(t,S)$ does not change during the interval $t \le \tau \le T$. In fact, it will change as new price information is observed. Our actual strategy will use only the initial instantaneous trade rate of this trajectory, continuously responding to price information. This is equivalently to following the strategy only for a very small time interval Δt, then recomputing. Thus our strategy is highly dynamic.

We shall mention that, according to this paper, the strategy determined is the true optimum strategy that would be computed by a full dynamic optimization. Loosely speaking, this will because the expected value of future updates is zero, and thus they do not change the strategy of a risk-neutral trader.

3.2. Trajectories by Calculus of Variations

We consider a small perturbation of the path $x(\tau) \to x(\tau) + \delta x(\tau)$ for $t \le \tau \le T$. Since $x(\tau)$ is fixed at time $\tau = t$ and $\tau = T$, this perturbation must have two conditions: $\delta x(t) = \delta x(T) = 0$. The associated trade rate is $\delta v(\tau) = -\delta x'(\tau)$, and the perturbation in cost (assuming that $x(\tau)$ and $\delta x(\tau)$ are twice differentiable) is

$$\begin{aligned} \delta E &= \eta \int_t^T 2v(\tau)\delta v(\tau) d\tau + \alpha_* \int_t^T \delta x(\tau) d\tau \\ &= \int_t^T (-2\eta x''(\tau) + \alpha_*) \delta x(\tau) d\tau \end{aligned} \tag{8}$$

where $\alpha_* = \alpha_*(t, S(t))$ is estimated in (2).

3.2.1 Unconstrained Trajectories

In this part, the authors temporarily neglected the sign constraint on $x'(\tau)$. Then $\delta x(\tau)$ may have either positive or negative values independently for each τ, and any optimal $x(\tau)$ must satisfy the ordinary differential equation (ODE)

$$x''(\tau) = \frac{\alpha_*}{2\eta}, \qquad t \le \tau \le T \tag{9}$$

The solution to this equation that satisfies the boundary conditions is

$$x(\tau) = \frac{T-\tau}{T-t} x(t) - \frac{\alpha_*}{4\eta} (\tau - t)(T - \tau), \quad t \le \tau \le T \tag{10}$$

and the corresponding instantaneous trade rate is

$$v(t,x) = -x'(\tau)|_{\tau=t} = \frac{x(t)}{T-t} + \frac{\alpha_*}{4\eta}(T-t)$$

as a function of time and shares remaining. This solution may violate the constraints: if α_* is large then the quadratic term in (10) may cause $x(\tau)$ to dip below zero, which would cause in (11) to become negative.

3.2.2 Constrained Trajectories

If the constraint becomes binding, then it is no longer obvious that the integration by parts procedure used to derived (8) is valid. For instance, if a trajectory that crosses the axis $x = 0$ is simply clipped to satisfy $x \geq 0$, then the derivative will be discontinuous. A more refined use of the calculus of variations gives the additional condition: $v(\tau)$ must be continuous (though not necessarily differentiable).

Thus when solutions meet the constraint, they must do so smoothly. Solutions are obtained by combining the ODE (9) in the regions of smoothness, with this "smooth pasting" condition at the boundary points.

The result can be summarized as follows. There is a critical value α_c such that

1. If $|\alpha_*| \leq \alpha_c$, then the constraint is not binding. The solution is as same as the unconstrained (given (10) and (11)).
2. If $\alpha_* > \alpha_c$, then the solution is the one of (10) and (11), with a

shortened end time $T_* < T$ determined by

$$T_* - t = \sqrt{\frac{4\eta x(t)}{\alpha_*}}$$

This value is determined such that $x'(T_*) = x'(T) = 0$. The threshold value α_c is the valued of α_* for which $T_* = T$:

$$\alpha_c(x(t), T-t) = \frac{4\eta x(t)}{(T-t)^2}$$

3. If $\alpha_* < -\alpha_c$, then the solution is the one of (10) and (11), except that trading does not begin until a starting time $t_* > t$ determined by

$$T - t_* = \sqrt{\frac{4\eta x(t)}{-\alpha_*}}$$

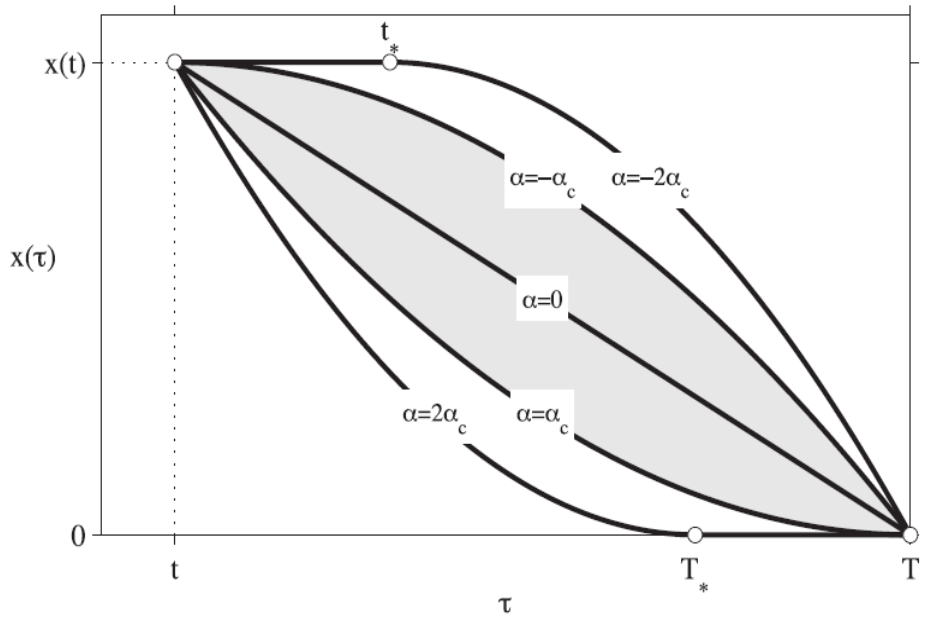
This value is determined such that $x'(t_*) = 0$ and $x(t_*) = x(t)$. The threshold value α_c is the value of $-\alpha_*$ for which $t = t_*$.

Then the overall trade rate formula may be summarized as

$$v(t,x,S) = \begin{cases} 0, & \alpha_* < -\alpha_c \\ \dfrac{x}{T-t} + \dfrac{\alpha_*}{4\eta}(T-t), & |\alpha_*| \le \alpha_c \\ \dfrac{x}{T_*-t} + \dfrac{\alpha_*}{4\eta}(T_*-t) = \sqrt{\dfrac{\alpha_* x}{\eta}}, & \alpha_* > \alpha_c \end{cases} \qquad (12)$$

This is the Bayesian adaptive strategy: a specific formula for the instantaneous trade rate as a function of price, time and shares remaining.

The following graph 1 shows the constrained solution $x(\tau)$ starting at time t with shares $x(t)$ and drift estimate α.



Graph 1

For $\alpha > 0$, the trajectories below the linear profile to reduce expected cost.

For $|\alpha| \leq \alpha_c$, the constrain is not binding (shaded region).

At $\alpha = \alpha_c$, the solutions become tangent to the line $x = 0$ at time $\tau = T$ and for larger values they hit $x = 0$ with zero shapes at $\tau = T_* < T$.

For $\alpha < -\alpha_c$, trading does not begin until time $\tau = t_* > t$.

3.3. Optimality of the Bayesian Adaptive Strategy

According to the paper, the Bayesian adaptive strategy (12) is "locally" optimal in the sense that at any intermediate time the authors used all the new information available and recomputed the trajectory for the remainder as though they would use the same

estimate until the end of trading. Since they do expect to update the estimate, it is not obvious that this is the true optimal strategy.

Using the methods of stochastic optima control, we can formulate a Hamilton-Jacobi-Bellman (HJB) partial differential equation (PDE) for the value function for the corresponding dynamic program.

Now we support our claim that the trade velocity (11) of the Bayesian adaptive strategy is in fact the optimal strategy for (7). For that, we will formulate the problem in a full dynamic programming framework.

The control, the state variables, and the stochastic differential equations of problem (7) are given by

$v =$ rate of buying

$x =$ shares remaining to buy $\quad dx = -v dt$

$\gamma =$ dollars spent so far $\quad d\gamma = (s + \eta v) v dt$

$s =$ stock price $\quad ds = \alpha dt + \sigma dB$

where $\alpha \sim N(\bar{\alpha}, \nu^2)$, chosen randomly at $t = 0$. We begin at $t = 0$ with shares $x(0) = X$, cash $\gamma(0) = 0$, and the initial stock price $s(0) = S$. The strategy $v(t)$ must be adapted to the filtration of B and must satisfy $x(T) = 0$. We focus on the unconstrained case and to not require $0 \le x(t) \le X$.

We want to find a control function $v(t)$ to minimize the final amount of dollars spent:

$$\min_{v(\tau)\ s.t.\ x(T)=0} E[\gamma(T)]\,.$$

This is a common problem in stochastic optimal control, and solved by dynamic programming. Standard techniques lead to the HJB PDE

$$u_t + \frac{1}{2}\sigma^2 u_{ss} + \alpha_* u_s + \min_v((su_\gamma - u_x)v + \eta u_\gamma v^2) = 0$$

for the value function

$$u(t,x,\gamma,s) = \min_{v(\tau),t\le\tau\le T\ s.t.\ x(T)=0} E[\gamma(T)]\,.$$

$\alpha_* = \alpha_*(t,s)$ denotes the estimate of α at time t as computed in (2). The optimal trade velocity is found as first order condition of $\min_v((su_\gamma - u_x)v + \eta u_\gamma v^2)$

$$v_*(t,x,\gamma,s) = \frac{u_x - su_\gamma}{2\eta u_\gamma} \tag{13}$$

and we have the final HJB PDE

$$u_t + \frac{1}{2}\sigma^2 u_{ss} + \alpha_* u_s - \frac{(su_\gamma - u_x)^2}{4\eta u_\gamma} = 0 \tag{14}$$

for $u(t,x,\gamma,s)$ together with the boundary condition

$$u(T,0,\gamma,s) = \gamma \quad \text{for all} \quad \gamma, s\,. \tag{15}$$

It is straightforward to check that

$$\begin{aligned} u(t,x,\gamma,s) = \gamma + xs + \frac{x^2\eta}{T-t} + \frac{x\alpha_*(t,s)(T-t)}{2} \\ - \frac{(T-t)^3\alpha_*(t,s)^2}{48\eta} - \int_t^T \frac{\sigma^2 v^4 (T-\tau)^3}{48\eta(\tau v^2 + \sigma^2)^2}\,d\tau \end{aligned} \tag{16}$$

satisfies the PDE (14) and the boundary condition (15).

Moreover, the corresponding optimal trade velocity (13) reads

$$v_*(t,x,\gamma,s) = \frac{x(t,x,\gamma,s)}{T-t} + \frac{\alpha_*(t,s)(T-t)}{4\eta}$$

which is exactly the trade velocity (11). That is, the Bayesian adaptive strategy is in fact the optimal strategy for minimum problem (7).

For the constrained case, the optimal velocity (13) becomes

$$v_*(t,x,\gamma,s)=\max\{\frac{u_x-su_\gamma}{2\eta u_\gamma},0\}$$

This makes the corresponding PDE even more highly nonlinear, and we do not know how to derive explicit solutions.

Furthermore, we may explicitly determine the gains due to adaptivity. The valued function for the dynamic strategy, at $t=0$ with initial shares X, may be written

$$E_{dyn}=E_{stat}-\Delta$$

where $E_{stat}=\frac{X^2\eta}{T}+\frac{X\bar{\alpha}T}{2}-\frac{T^3\bar{\alpha}^2}{48\eta}$ is the expected cost of the non-adaptive strategy determined at $t=0$ using the prior expected drift $\bar{\alpha}$. The additional term is determined as

$$\Delta=\frac{\sigma^2T^2}{48\eta}\int_0^1\frac{(1-\delta)^3}{(\delta+\rho)^2}d\delta,\ \ \rho=\frac{\sigma^2}{\nu^2T},\ \delta=\frac{t}{T}$$

which is the reduction in expected cost obtained by using the Bayesian adaptive strategy (note that $\Delta>0$).

The gain Δ is independent of initial portfolio size X and thus, as discussed above, represents the gains from a proprietary trading strategy superimposed on the risk neutral liquidation profile. It can be seen that $\Delta\sim O(T^4)$ when T is small and $O(T^2)$ when T is

large, so the adaptivity adds very little value if applied to short-term correlation. This accounts for the small gains obtained by Bertsimas and Lo (1998) as discussed by Almgren and Chriss (2000).

For the constrained case, the HJB equation has complicated boundary conditions and we are unable to determine an explicit analytic solution. However, we do not believe that imposition of the constraint should not change the relation between the static and the dynamic solutions. Thus we believe that our Bayesian dynamic solution is the dynamic optimal solution in the constrained case as well.

4. Conclusion

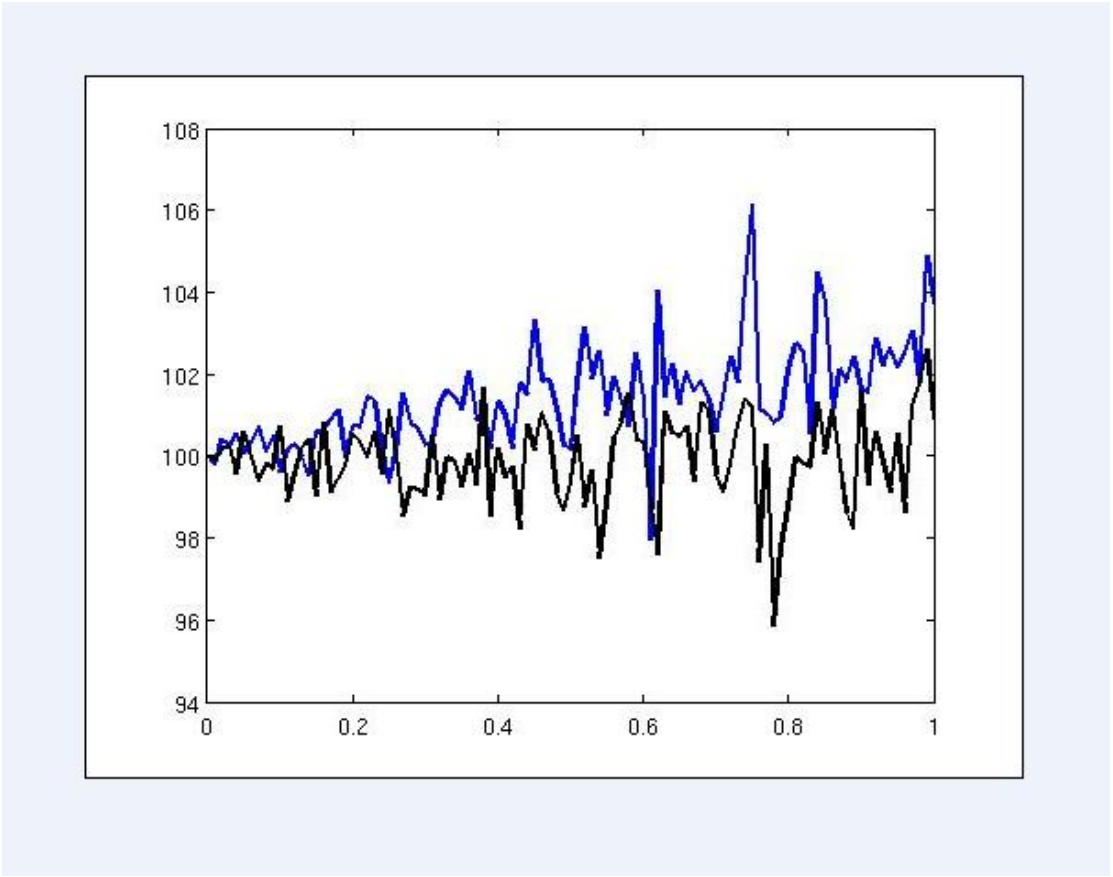
We have presented a simple model for momentum in price motion based on daily trading cycles, and derived optimal risk neutral adaptive trading strategies. The momentum is understood to arise from the correlated trade targets being executed by large institutional investors. The trader begins with a belief about the direction of this imbalance and expresses a level of confidence in this belief that may range anywhere from perfect knowledge to no knowledge. This belief is then updated using observations of the price process during trading. Under the assumptions of the model, the solutions deliver better

performance than non-adaptive strategies.

Moreover, we will show some results of the strategies computed by this method. To produce these graphs, we began with a prior belief for α having mean $\bar{\alpha} = 0.7$ and standard deviation $\nu = 1$. For each trajectory, we generated a price path $S(t)$ for $0 \le t \le 1$ with volatility $\sigma = 1.5$. We also set the impact coefficient $\eta = 0.07$ and the initial shares $X = 1$, meaning that liquidation the holdings using VWAP across one day will incur a realized price impact of 7 basis points. And the initial stock price $S_0 = S(0) = \$100$.

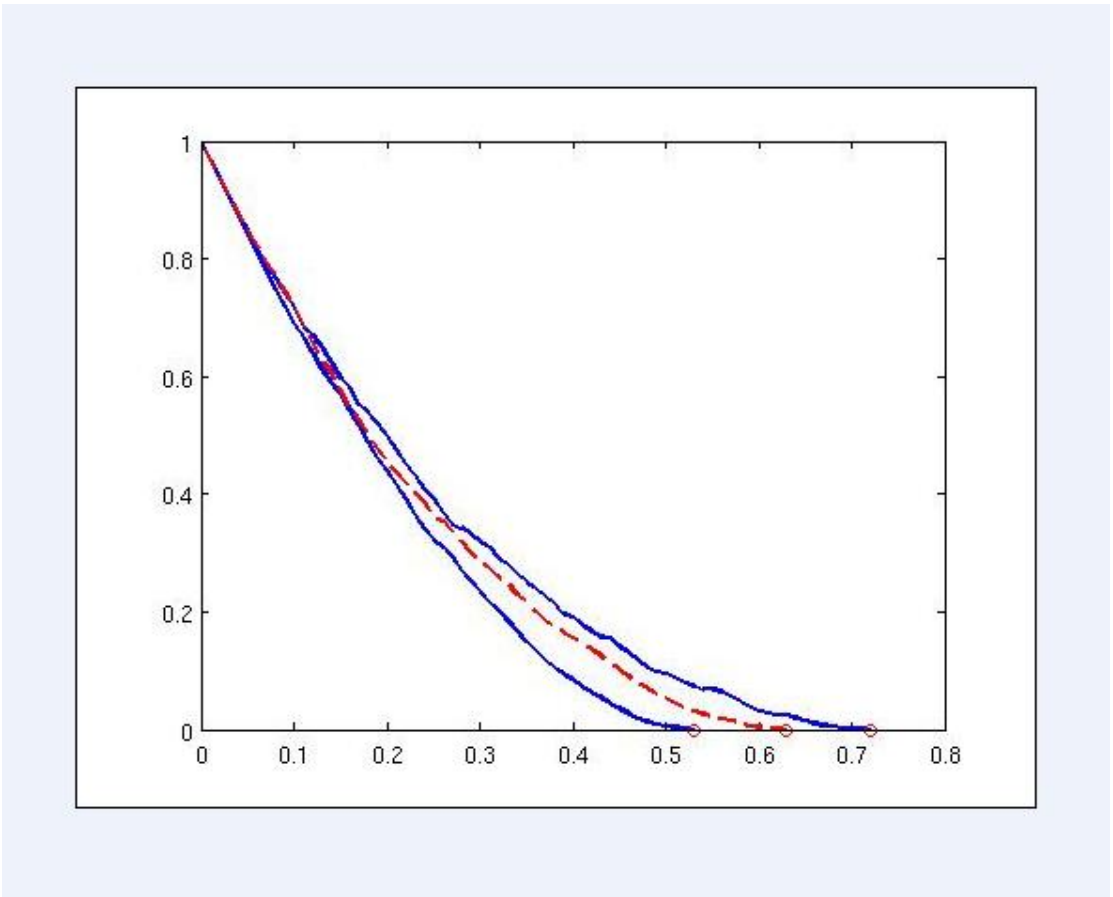
Then for each sample path, we evaluate the share holdings $x(t)$ using the Bayesian update strategy (12) and plot the trajectories. For comparision, we also show the optimal static trajectory using only the initial estimate of the drift.

For these initializations, it is better to change the setup and get different pictures to compare. But here, we only give two pictures, which are stock price simulation and the corresponding trajectories, respectively, using the above initialization.



Graph 2

Graph 2 shows two stock price sample paths using the above prior belief. Axis x is the time horizon from 0 to 1, axis y is the stock price $S(t)$.



Graph 3

Graph 3 is the corresponding trajectories. The red line represents the optimal static trajectory. The lower blue one is corresponding with the blue stock price path and the upper one is according to the black one. In both, the price initially trends downward (more strongly for the blue one), causing the trader to estimate a drift is smaller than his prior belief and to slow down his trading relative to the static solution.

5. Difficulties and Justification

In the basis of article Almgren and Lorenz (2006), we have known that it is very difficult to justify the model by empirical data. Because in this model, the random daily drift is superimposed on the price volatility caused by small random trader. In theory, these two sources of randomness can be eliminated by measuring volatility on an intraday time scale and comparing it to daily volatility. If daily volatility is higher than intraday, then the difference can be attributed to serial correlation of the type considered here. But in practice, because real price processes are far from Gaussian, it is difficult to do this comparison with any degree of reliability, even if one restricts attention to days when there is large institutional flow.

Therefore, we strongly believe that the difficulties and only just followed the methodology to simulate the optimal trading strategy. For further study, we should try these empirical improve this method for practical use.

6. Appendix

Here we show our MATLAB programming codes.

```
close all;
clear all;


% initialize
alpha_mean = 0.7;
s0 = 100;
std_v = 1;
sigma = 1.5;
T = 1;
time_step = 100;
dt = T / time_step;

yeta=0.07;



% genetate stock price
alpha(1:2)=alpha_mean+std_v*randn(1,2);
alpha(3)=alpha_mean;
t=0:dt:T;
for num=1:3

    X=ones(time_step+1,1);
    Y=X(1);
    s=s0+alpha(num)*t+sigma*sqrt(t).*randn(1,time_step+1);

    figure(1);
    if num==1
    plot(t,s,'b','linewidth',2);hold on;
    elseif num==2
    plot(t,s,'black','linewidth',2);hold on;
    end

    alpha_star =
(alpha_mean*sigma^2+std_v^2*(s-s0))./(sigma^2+std_v^2*t);
    for n=1:time_step
        alpha_c=4*yeta*Y/(T-t(n))^2;
        if alpha_star(n)<-alpha_c
            v(n)=0;
```

```
        elseif alpha_star(n)>alpha_c
              v(n)=sqrt(alpha_star(n)*X(n)/yeta);
        else

v(n)=X(n)/(T-t(n))+alpha_star(n)/(4*yeta)*(T-t(n));
        end
        Y=Y-v(n)*dt;
        X(n+1)=Y;
        if Y<1e-6
           break;
        end
    end
    figure(2);
    m=0:dt:n*dt;
    if  num<3
    plot(m,X(1:n+1),'linewidth',2); hold on;
    plot(n*dt,X(n+1),'ro');
    ylim([0,1]);
    else
    plot(m,X(1:n+1),'--r','linewidth',2); hold on;
    plot(n*dt,X(n+1),'ro');
    ylim([0,1]);
    end
end
```

Acknowledgement

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