

Department of Mathematics

QF5205 Topics in quantitative finance

Group Project Report

Predictability of Non-Linear Trading Rules in the US Stock Market

Chong & Lam 2010

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Abstract

Most of the existing technical trading rules are linear in nature. We will investigate the predictability of nonlinear time series model based trading strategies in the HONG KONG and SINGAPORE stock market. We analyze the performance of the nonlinear trading rule by comparing two predictions based on independent stock market. In this report we use a two-regime SETAR (1) model, which fits for HONG KONG stock market better than the SINGAPORE stock market. We also use this model to make prediction with different observation windows. It is found that as the observation window increases, the fraction of positive buy and sell decreases, and when the observation window is 150, a highest buy-sell return is generated by the SETAR rules.

Introduction

For a long time the topic “whether technical analysis for trading is useful” has been discussed. In 1966 Fama and Blume are the first to do academic study on the performance of technical trading rules. In 1970 Jensen and Bennington followed to investigate the performance of filter rules in the U.S. stock market and find that they fail to beat the buy-and-sell strategy. In 1992 Brock does academic research to find evidence to support for the effectiveness of the moving average, and he also found the trading range break rules in the Dow Jones Industrial Average Index. While most of the early studies focus on simple trading rules, there has been a growing interest in the performance of nonlinear trading rules in recent years. In the original paper, they do research on three models: self-exciting threshold autoregressive (SETAR) model, autoregressive (AR) model and variable moving average (VMA) model. Comparing the three models we find that the moving average model has the worst performance and AR model is just a special case of SETAR model, so we focus on the SETAR model in our project. In light of the growing popularity of nonlinear trading rules and the increasing documentation of stock-return asymmetry, we attempt to investigate the predictability of trading strategies based on nonlinear models. We will compare trading strategies generated by the self-exciting threshold autoregressive (SETAR) model with different observation windows. The rest of this paper is organized as follows. Section 2 presents the models and the trading strategies. Section 3 describes the data and test statistics. Section 4 reports the empirical results. Section 5 concludes and summarizes our results.

Literature Review

3 trading models (the Variable Moving Average Model (VMA), the Autoregressive Model (AR) and the Self-Exciting Threshold Autoregressive Model (SETAR)) with their results on four major US indices (the Dow Jones Industrial Average (DJIA), NASDAQ Composite Index (NASDAQ), New York Stock Exchange Composite Index (NYSE) and Standard and Poor 500 Index (S&P 500)), were analyzed in the original paper.

Variable Moving Average Model (VMA)

The VMA is just the simple moving average crossover strategy, where the value of an n day moving average is calculated as below and P_m is the m^{th} day closing price.

$$MA = {\sum_{m-n}^{m} P_m}/{n}$$

The general form of a VMA is given as VMA(S, L), where S represents the number of days in the short term window, and L represents the number of days in the long term window.

The trading rule is then defined as

Buy if MA (S) > MA (L)

Sell if MA (S) < MA (L)

Autoregressive Model (AR)

The AR model is a linear time series model, often being referred to as the AR (p) model, and given by

$$\Delta \hat{Y}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \Delta Y_{t-i} + \varepsilon_t$$

Where $\Delta\hat{Y}_t$ is the predicted natural logged difference of the t^{th} day stock index

α_0 is the constant term of the linear regression

α_i is the coefficient of the t-i^{th} day logged difference of the stock index

p represents the order of autoregressive parts, that is, the number of dependent variables. For example, if p = 2, then $\Delta\hat{Y}_t$ is dependent on both ΔY_{t-1} and $\Delta\hat{Y}_{t-2}$.

ε_t is the residual error

For the paper, an AR (1) model was used.

$$\Delta \hat{Y}_t = \alpha_0 + \alpha_1 \Delta Y_{t-1} + \varepsilon_t$$

The trading rule is defined as

Buy if $\Delta \hat{Y}_t^w > 0$,

Sell if $\Delta \hat{Y}_t^w < 0$,

where $\Delta\hat{Y}_t^w$ is the predicted return estimated from the regressed model based on the most recent w observations.

Self-Exciting Threshold Autoregressive Model (SETAR)

The most exciting model in the paper, the SETAR model is a non-linear time series model that is an extension of the AR model. It enables a higher degree of flexibility due to the inclusion of a threshold parameter, a 'switch' for the model to go into different regimes under different conditions.

The SETAR model is generally referred to as a SETAR (k, p) model, given as

$$\Delta\hat{Y}_t = \alpha_{0,j} + \sum_{i=1}^{p} \alpha_{i,j}\,\Delta Y_{t-i} + \varepsilon_t \quad if\ \gamma_{j-1} < \Delta Y_{t-d} \leq \gamma_j \quad for\ j = 1, j = 2, \ldots j = k$$

Again, $\Delta\hat{Y}_t$ is the predicted natural logged difference of the t^{th} day stock index

$\alpha_{0,j}$ is the constant term of the linear regression for the j^{th} regime

$\alpha_{i,j}$ is the coefficient of the t-i^{th} day logged difference of the stock index for the j^{th} regime

p represents the order of autoregressive parts mentioned in the previous section

γ_j is the threshold parameter for the j^{th} regime, and $\gamma_0 = -\infty$, $\gamma_k = +\infty$

d is the delay parameter

ε_t is the residual error

Data Selection & Calculations

Data Selection

Our study encompasses two major Asian indices, The HANG SENG INDEX, and the STRAITS TIMES INDEX, with their respective data downloaded from yahoo finance.

For The Hang Seng Index, the data comprises of daily closing prices from 31st Dec 1986 to 31st Dec 2010, giving a total of 5962 observations. Whereas for The Straits Times Index, a total of 5754 observations were collected from the daily closing prices between 31st Dec 1987 to 31st Dec 2010.

Statistical Studies on Selected Data

The table below summarizes the various calculations and statistical tests performed on the observations.

	Hang Seng index	Straits Times Index
Count	5961	5753
Mean	0.000368024	0.000233276
Standard Error	0.000232311	0.000172286
Standard Deviation	0.017936129	0.013067676
Sample Variance	0.000321705	0.000170764
Kurtosis	56.6643583224#	8.484925539#
Skewness	-2.448071427#	-0.121755714#
JB stat	721240.1595#	7225.697523#
ρ(1)	0.018987413	0.087835121
ρ(2)	-0.022648541	0.036533523
ρ(3)	0.057481869	-0.000380175
ρ(4)	-0.019593225	0.000678288
ρ(5)	-0.030011843	-0.002237887
Q(5)	32.58874337#	52.12378136#

Table 1 Summary statistics for daily returns- full sample.
Returns are defined as the log difference of the stock index value.
"JB stat" represents the Jarque-Bera test for normality.
ρ(i) is the estimated autocorrelation at lag i.
Q(5) is the Ljung-Box Q statistics at lag 5
Numbers marked with # are significant at the 1% level.
And the calculation for the below statistics are shown in Appendix A

1. skewness, kurtosis
2. Jarque-Bera statistics
3. Ljung and Box Q statistic

From the values of skewness, kurtosis, and Jarque-Bera statistics, it can be concluded that returns are skewed, leptokurtic, and not normally distributed.
The Ljung-Box Q statistics at 5^{th} lag is significant at 1%, which is suggestive of substantial serial correlation in stock returns. This is essential for the existence of trading rule profits.
These results coincide with that found in the research paper, which hopefully, provide some positive indication of SETAR model's performances for our project.

Statistics

On each trading day, a position of buy or sell is taken. The conditional mean and variance of buy (sell) returns can be respectively written as follows:

$$\pi_{b(S)} = \frac{1}{N_{b(S)}} \sum_{t=1}^{N} \Delta Y_{t+1} I_t^{b(s)} \qquad \sigma^2_{b(s)} = \frac{1}{N_{b(S)}} \sum_{t=1}^{N} (\Delta Y_{t+1} - \pi_{b(s)})^2 I_t^{b(s)}$$

where $\pi_{b(S)}$ stands for the mean return of buy (sell) periods, $\sigma^2_{b(s)}$ refers to the conditional variance of buy (sell) signals, $N\ b(s)$ represents the number of buy (sell) days, N is the number of observations of the whole sample, ΔY_{t+1} is the one-day holding period return, and $I_t^{b(s)}$ is an indicator function which equals one if a buy (sell) signal is observed at time t, and equals zero otherwise. We first test for the difference between the mean buy (sell) return and the one-day unconditional mean.

The null and the alternative hypotheses are respectively:

$$H_0: \quad \pi_{b(s)} = \bar{\pi} \qquad H_1: \quad \pi_{b(s)} \neq \bar{\pi}$$

And the conventional t-ratio for the mean buy (sell) return is

$$t_{b(s)} = \frac{\pi_{b(s)} - \bar{\pi}}{\left[\frac{\sigma^2}{N_{b(s)}} + \frac{\sigma^2}{N} \right]^{\frac{1}{2}}},$$

where $\bar{\pi}$ is the unconditional one day mean and σ^2 is the unconditional variance.

For the buy-sell spread, the null and the alternative hypotheses can be expressed as follows.

$$H_0: \quad \pi_b - \pi_s = 0 \quad H_1: \quad \pi_b - \pi_s \neq 0$$

The t-statistic is:

$$t_{b-s} = \frac{\pi_b - \pi_s}{\left[\frac{\sigma^2}{N_b} + \frac{\sigma^2}{N_s} \right]^{\frac{1}{2}}}$$

Parameter Estimation

In our project, we use the two-regime SETAR (1) model:

$$\Delta Y_t = (\alpha_0 + \alpha_1 \Delta Y_{t-d}) I[\Delta Y_{t-d} \geq \gamma] + (\beta_0 + \beta_1 \Delta Y_{t-d}) I[\Delta Y_{t-d} < \gamma] + \varepsilon_t,$$

where $\Delta Y_t = Y_t - Y_{t-1}$ denotes the first difference of the natural logarithm of the stock index at time t, γ represents the threshold value, d refers to the delay parameter, and I[A] is an indicator function that equals 1 if condition A is satisfied, and equals zero otherwise. Thus, we may find that our model is just the combination of two linear AR(1) models.

Given a fixed observation window with w observations $\Delta Y_1, \Delta Y_2, ..., \Delta Y_w$, we use Ordinary Least Square (OLS) method to determine the parameters, namely γ, α_0, α_1, β_0 and β_1. This method is developed by Bruce E. Hansen in 1997 in his paper"Inference in TAR models". We first define a two by one vector $X_t = (1 \quad \Delta Y_{t-1})^T$, four by one vector $X_t(\gamma) = (X_t^T I_{\{\Delta Yt-d \geq \gamma\}} \quad X_t^T I_{\{\Delta Yt-d \leq \gamma\}})^T$ and the parameter vector $\theta = (\alpha_0 \quad \alpha_1 \quad \beta_0 \quad \beta_1)^T$. Then the model is transformed into $\Delta Y_t = X_t(\gamma)^T \theta + \varepsilon_t$. Then by the OLS method, the estimate of θ is

$$\hat{\theta}(\gamma) = \left(\sum_{t=1}^{n} x_t(\gamma) x_t(\gamma)' \right)^{-1} \left(\sum_{t=1}^{n} x_t(\gamma) y_t \right),$$

which is a function of γ. The residual $e_t(\gamma) = \Delta Y_t - X_t(\gamma)^T \theta(\gamma)$ and the residual variance is

$$\hat{\sigma}_n^2(\gamma) = \frac{1}{n} \sum_{t=1}^{n} \hat{e}_t(\gamma)^2$$

. In our case, we fix d = 1. According to the paper, d is from 1 to 5. However, in our case, when d = 1, the model has the best performance, i.e. the residual variance attains its minimum value. Thus, we take d = 1. Observe that the residual variance only takes on at most w different values as γ varies, where n is our sample size. We set γ equal to ΔY_{t-1}, t = 2,…,w. Then grid search γ which minimizes the sum of squared residuals of the model. Finally substitute the optimal γ into the estimate of θ to obtain

$$\hat{\theta}(\gamma) = \left(\sum_{t=1}^{n} x_t(\gamma) x_t(\gamma)' \right)^{-1} \left(\sum_{t=1}^{n} x_t(\gamma) y_t \right)$$

Thus, we can determine the parameter α_0, α_1, β_0, β_1 based on the given observation window.

Trading Strategy

According to the SETAR model, the trading strategy is as following:

Buy if $E[\Delta Y_t^w] > 0$

Sell if $E[\Delta Y_t^w] < 0$

where $E[\Delta Y_t^w]$ is the conditional expectation of ΔY_t based on the most recent w observations and w is our observation window. When we apply the trading strategy, just imagine that we use the model to predict the price tomorrow. If tomorrow's price is higher than today, we buy. Otherwise, we sell.

Following the paper, we employ the recursive technique to generate the SETAR one-step-ahead forecast series. The algorithm is as follows:

Given the observation window w and sample size n, and set i = 1,

1. Use $\{\Delta Y_i, ..., \Delta Y_{i+w-1}\}$ to compute the fitted parameters α, β, γ
2. Use the parameters obtained in Step 1 to calculate fitted $\Delta\hat{Y}_{i+w}$.
3. Apply the trading strategy. Set "Buy" or "Sell" signal to the $(i + w)^{th}$ day according to $\Delta\hat{Y}_{i+w}$.
4. Set i = i + 1 and then repeat Step 1 to 3.

When using this algorithm, we may notice that the value of α and β change with the observation window, as we use the most recent w observations. So as we move, we roll the window forward and update α and β to get the next prediction of ΔY. Thus, for each trading day, we set buy signal or sell signal (except for the first w days).

Empirical results

We use self-exciting threshold autoregressive (SETAR) model to generate the results and compare with different observation windows for the two different stock indices.

	Hang Sang Index	Straits Times Index
SETAR(1) parameter estimates		
$\Delta Y_t = (\alpha_0 + \alpha_1 \Delta Y_{t-1}) I_{\{\Delta Yt-d \geq \gamma\}} + (\beta_0 + \beta_1 \Delta Y_{t-1}) I_{\{\Delta Yt-d \leq \gamma\}}$		
α_0	-0.001866	0.002537
α_1	0.159026	0.15324
β_0	0.001560	-0.000973
β_1	0.071926	0.14072
γ	-0.02705	-0.017934
d	1	1

Table 2. Parameter estimation of SETAR(1) model.

Table 2 shows the parameter estimates for SETAR(1) model. The parameters are estimated by OLS. Given a one-period delay, we select the threshold value that gives the smallest residual sum of squares. ΔY_t is the continuously compounded return on day t, d is the delay parameter, and γ is the threshold value. All numbers are statistically different from zero.

SETAR rules are respectively identified as SETAR(P, W), where P is the AR order and W is the observation window.

'N(Buy)' and 'N(Sell)' are the number of buy and sell signals.

'σ (Buy)' and 'σ(sell)' are the standard deviation of buy and sell periods.

'Buy>0' and 'Sell>0' are the fraction of positive buy and sell returns, i.e. the probability we predict the trend correctly.

'Buy', 'Sell' and 'Buy - Sell' show the one-day conditional mean for buy, sell and buy-sell returns, where

$$\text{Buy} = \frac{1}{N(Buy)} \sum_{i=1}^{N(Buy)} \Delta \hat{Y}_i \, I_{\{\Delta Y \text{ with buy signal}\}},$$

$$\text{Sell} = \frac{1}{N(Sell)} \sum_{i=1}^{N(Sell)} \Delta \hat{Y}_i \, I_{\{\Delta Y \text{ with sell signal}\}}$$

and "Buy – Sell" is just Buy + |Sell|, which is the expected profit.

Numbers in parentheses are t-ratios testing the significance of the mean buy return from the unconditional mean, the mean sell return from the unconditional mean, and the buy-sell

spread from zero.

Numbers marked with # are significant at the 1% level.

The following table represents for the empirical results of implementing the trading strategies on the HANG SENG INDEX

Trading rule	**N (Buy)**	**N (Sell)**	**Buy**	**Sell**	**σ (Buy)**	**σ(Sell)**	**Buy>0**	**Sell>0**	**Buy-Sell**
SETAR (1, 50)	3220	2690	0.023596 (6.4727)#	-0.073722 (-7.2962)#	0.13314	0.43366	0. 5341	0. 4838	0.097318 (11.9253)#
SETAR (1, 150)	3075	2735	0.029190 (12.0874)#	-0.123009 (-10.5941)#	0.07133	0.45865	0.5221	0.4737	0.152199 (17.6622)#
SETAR (1, 200)	3086	2672	0.033439 (14.2012)#	-0.103811 (-15.6311)#	0.07675	0.26494	0.5168	0.4667	0.137250 (25.8362)#

Table 3(a). Empirical results of implementing the trading strategies on the Hang Seng Index.

Trading rule	**N (Buy)**	**N (Sell)**	**Buy**	**Sell**	**σ(Buy)**	**σ(Sell)**	**Buy>0**	**Sell>0**	**Buy-Sell**
SETAR (1, 50)	3059	2643	0.027662 (6.1744)#	-0.031135 (-6.5936)#	0.09069	0.27411	0.5343	0.5023	0.058797 (11.0532)#
SETAR (1, 150)	2846	2756	0.033272 (5.8103)#	-0.259817 (-6.0003)#	0.09172	1.52002	0.5325	0.4998	0.293089 (10.1728)#
SETAR (1, 200)	2911	2641	0.033382 (5.6593)#	-0.208770 (-6.0386)#	0.08633	1.27458	0.5239	0.4915	0.242152 (10.1307)#

Table 3(b). Empirical results of implementing the trading strategies on the Straits Times Index.

Tables 3(a) and (b) present the results of the trading rules on the two stock indices. The SETAR rules are denoted as SETAR (P, W), where P represents the AR order and W represents the window width. Columns 2 and 3 report the number of Buy and Sell signals. Columns 8, 9 and 10 show the one-day conditional mean for buy, sell and buy-sell returns.

Table 3(a) contains the empirical results of implementing the trading rule on the HANG SENG INDEX series. The results are encouraging. The t-statistics in the "Buy – Sell" column suggests that rules with all three observation windows produce significant "buy – sell" returns at 1% level. Moreover, the highest attainable buy – sell return is 15.2199% from SETAR (1, 150) trading rules. According to column 8 and 9, the predictability of "buy" signal is better than "sell" signal in all three SETAR trading rules.

Table 3(b) reports the results for the Straits Times Index series. All three trading rules are significant at the 1% level. The highest attainable buy – sell return is 29.3089% from SETAR (1, 150) trading rules. Consistent with the results of the HANG SENG INDEX sample, we find that SETAR (1, 150) is the most rewarding rule. According to column 8 and 9, the predictability of "buy" signal is better than "sell" signal in all three SETAR trading rules.

In sum, the SETAR model performs well in both Hong Kong and Singapore stock market. According to both tables we find that as the observation window increases, the fraction of positive "buy" and "sell" decrease. Moreover, in both HANGSENG INDEX and STRAITS TIMES INDEX, the fraction of positive "buy" is higher than the fraction of positive "sell", and SETAR (1, 150) is the most rewarding rule.

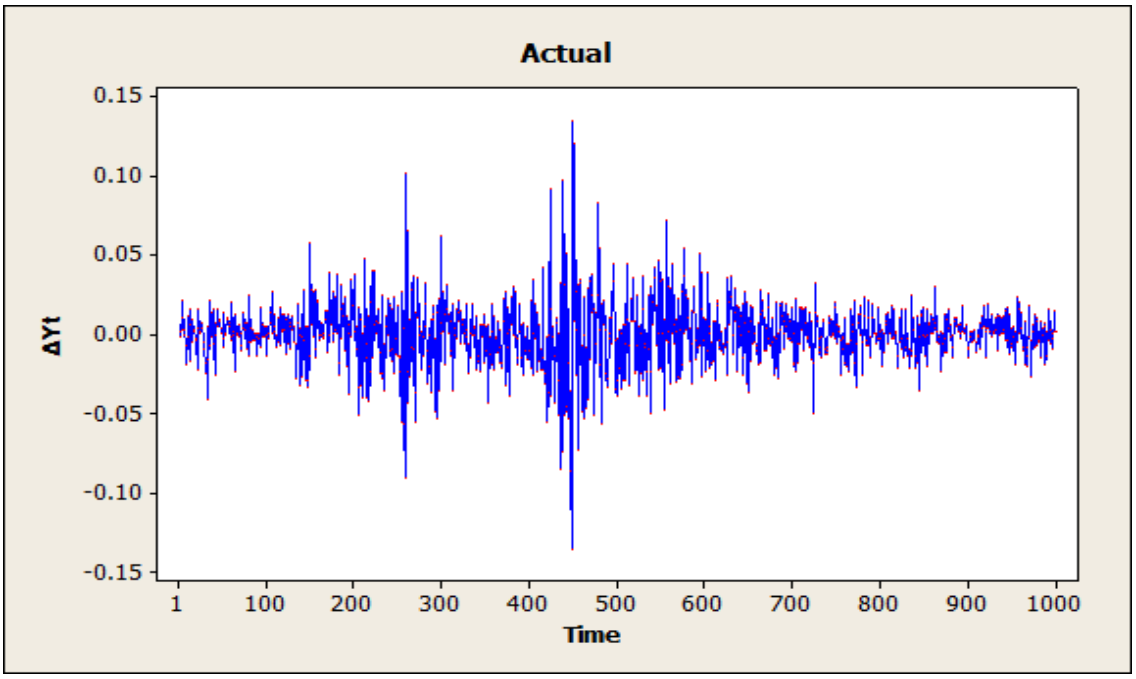


Fig. 1(a) ΔY_t of HANGSENG INDEX

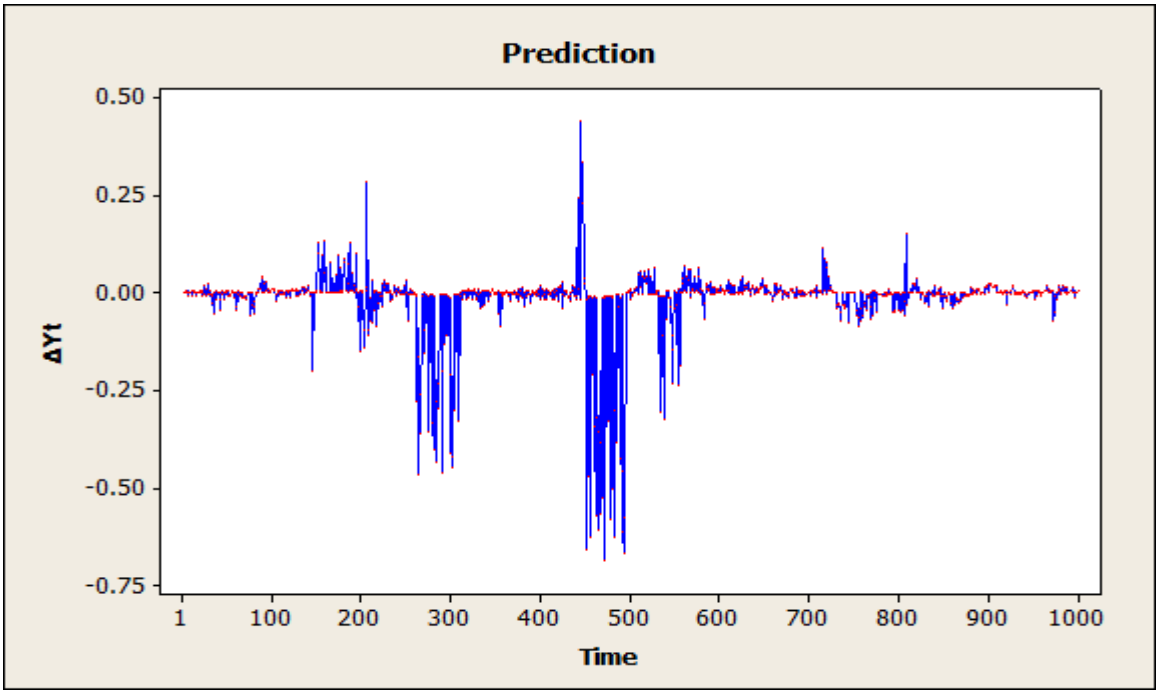


Fig. 1 (b) the predicted ΔY_t of HANGSENG INDEX

Figure 1(a) and Figure 1(b) show the real ΔY_t and predicted $\Delta \hat{Y}_t$ with observation window 50 of HANGHENG INDEX, respectively. We may find that the predicted time series generally capture the special pattern of original data. However, the predicted values may not be accurate enough to obtain an accurate expected profit.

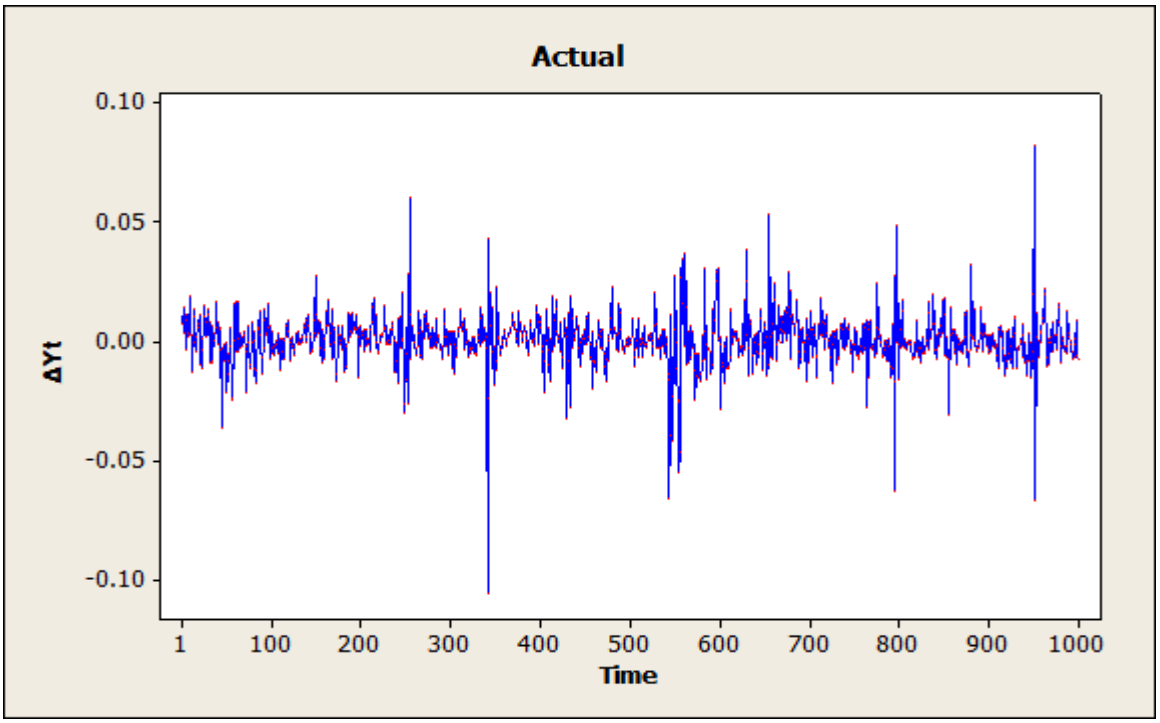


Fig. 2(a) ΔY_t of STRAITS TIMES INDEX

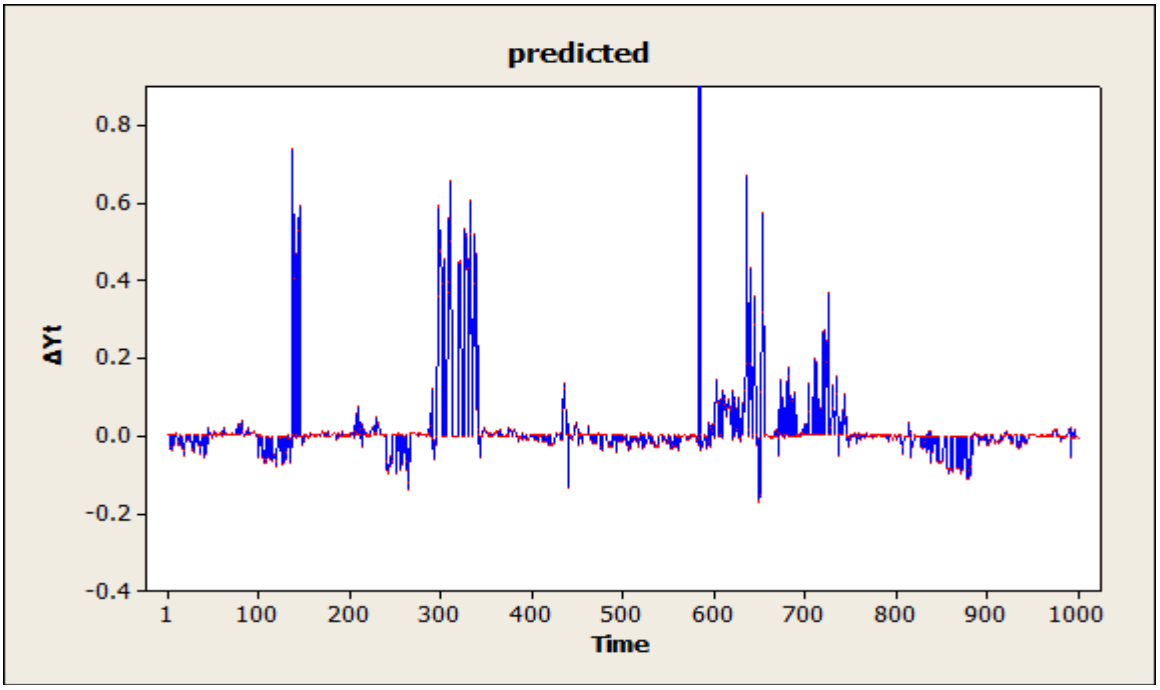


Fig. 2(b)) the predicted ΔY_t of STRAITS TIMES INDEX

Figure 2 (a) and Figure 2(b) show the real ΔYt and predicted ΔYt with observation window 50 of STRAITS TIMES INDEX, respectively. Compared with the prediction of HANGSENG INDEX, the predicted time series for STRAITS TIMES INDEX dose not capture the pattern of data very well. Thus, the two-regime SETAR(1) fits HANGSENG INDEX better than STRAITS TIMES INDEX.

Conclusion

This report investigates the prediction performance with trading rules based on the non-linear self-exciting threshold autoregressive (SETAR) model, which can be found in the research paper, “Predictability of Non-Linear Trading Rules in The US Stock Market”, by Chong & Lam 2010. We use the model to make prediction on the Hong Kong HANG SENG INDEX and the Singapore Straits Times Index. The SETAR model used in our project, after analyzing the research paper and doing our own statistical analysis, is a two regime SETAR (1) model, where ‘1’ represents the order of the autoregressive part. Statistical tests indicate that the difference of natural logarithm of daily stock closing prices are skewed, leptokurtic, and non-normally distributed. Ljung-Box Q statistics at 5^{th} lag suggests substantial serial correlation, which is essential for trading returns.

Comparing the two stock market index we find that the SETAR trading rule fits the HANG SENG INDEX better. And as the observation window increases the fraction of positive buy and sells decreases, the highest buy-sell return is got at observation window 150 for both stock markets.

Acknowledgements

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Appendix A

1. The skewness and kurtosis of ΔYt are defined as

$$S(\Delta Y) = E[\frac{(\Delta Y - \mu)^3}{\sigma^3}]$$

$$K(\Delta Y) = E[\frac{(\Delta Y - \mu)^4}{\sigma^4}]$$

To test the skewness of the returns, we consider the null hypothesis

H0: S(r) =0versus the alternative hypothesis

Ha: S(r) ≠ 0

The r ratio statistic of the sample Skewness in is:

$$t = \frac{\hat{S}(\Delta Y)}{\sqrt{\frac{6}{N}}}$$

Test the excess kurtosis of the return series using the hypotheses

H0: K(r)-3 = 0

Ha: K(r)-3 ≠ 0

The test statistic is

$$t = \frac{\hat{K}(\Delta Y) - 3}{\sqrt{\frac{24}{N}}}$$

which is the asymptotically standard normal random variable. The decision rule is to reject H0 if and only of the p value of the test statistics is less than the significance level α

2. The Jarque and Bera(JB) combine the two prior tests and use the test statistics

$$JB = \frac{\hat{S}^2(\Delta Y)}{\frac{6}{N}} + \frac{[\hat{K}(\Delta Y) - 3]^2}{\frac{24}{N}}$$

Which is asymptotically distributed as a chi squared random variable with 2 degrees of freedom, to test for the normality ofΔY . One rejects Ho if normality if the P value of the JB statistic is less than the significance level.

3. Ljung and Box Q statistic

$$Q(m) = T(T+2)\sum_{l=1}^{m} \frac{\hat{\rho}_l^2}{T-l}$$

As a test for the null hypothesis H0: $\rho_1 = \ldots = \rho_m$ against the alternative hypothesis Ha: $\rho_i \neq 0$ for some i={1,…m}. Q statistic is asymptotically a chi-squared random variable with m degrees of freedom.

The decision rule is to reject H0 if $Q > X_{\alpha}^{2}$ where X_{α}^{2} denotes the 100(1-α)th percentile of a chi-squared distribution with m degrees of freedom.